Repetitive processes

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Discrete linear repetitive processes

- Let $\alpha < \infty$ denote the (constant) pass length
- Let $0 \geq p \geq \alpha - 1$ be a point on each pass k, $k \geq 0$
- Let
  - $x_k(p) \in \mathbb{R}^n$ - state vector
  - $y_k(p) \in \mathbb{R}^m$ - output vector
  - $u_k(p) \in \mathbb{R}^l$ - input vector
  - $d_l \in \mathbb{R}^n$ - vector with constant entries
  - $y(p) \in \mathbb{R}^m$ - vector, whose entries are known functions of p

State space model of linear repetitive process

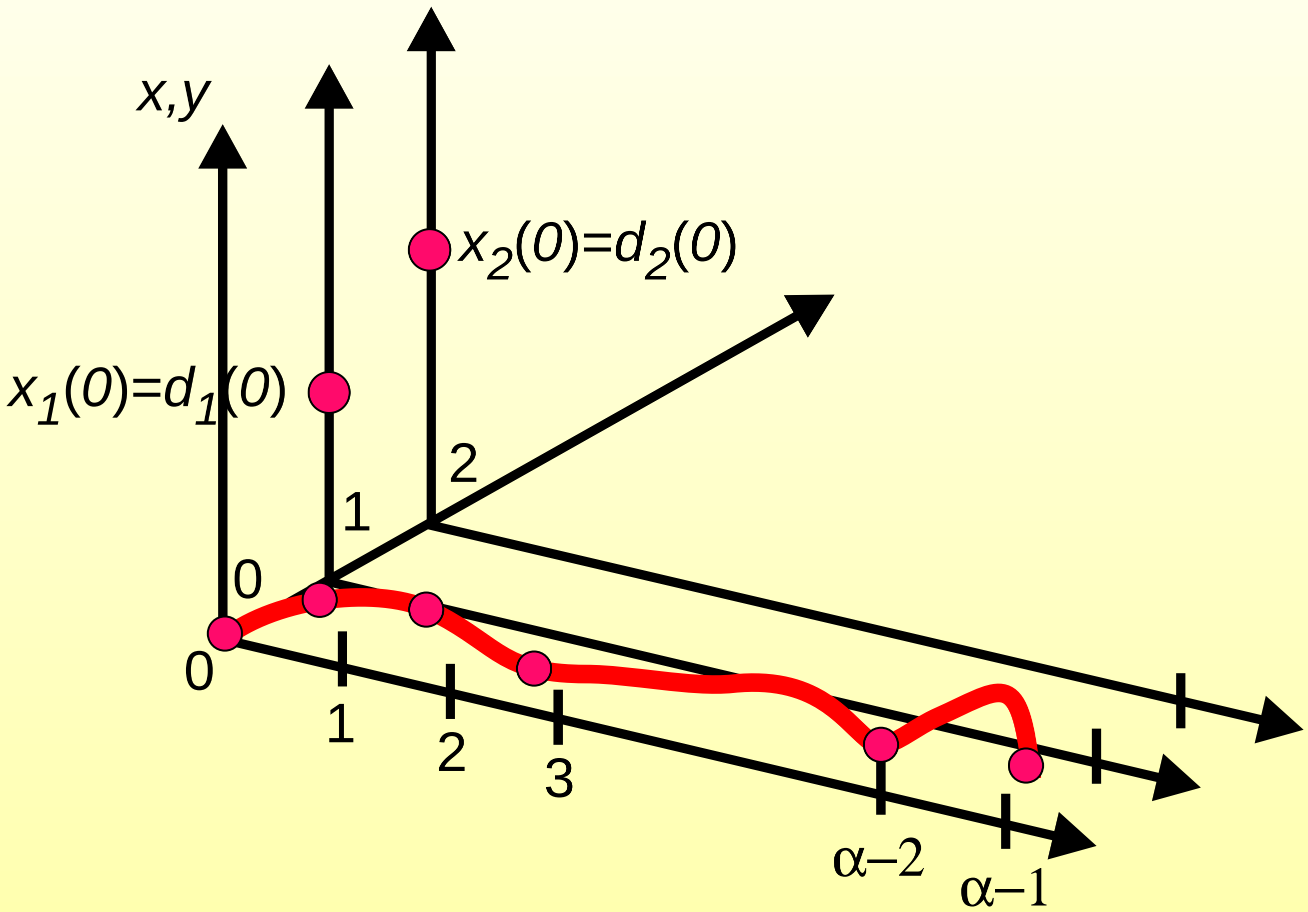
$$\begin{aligned} x_{k+1}(p+1) &= A\,x_{k+1}(p) + Bu_{k+1}(p) + B_0 y_k(p) \\ y_{k+1}(p) &= C\,x_{k+1}(p) + Du_{k+1}(p) + D_0 y_k(p) \end{aligned}$$

where A, B, B_0, C, D, D_0 are matrices of appropriate dimensions: $\mathbb{R}^{n\times n}$, $\mathbb{R}^{n\times l}$, $\mathbb{R}^{n\times m}$, $\mathbb{R}^{m\times n}$, $\mathbb{R}^{n\times l}$, $\mathbb{R}^{m\times m}$ respectively

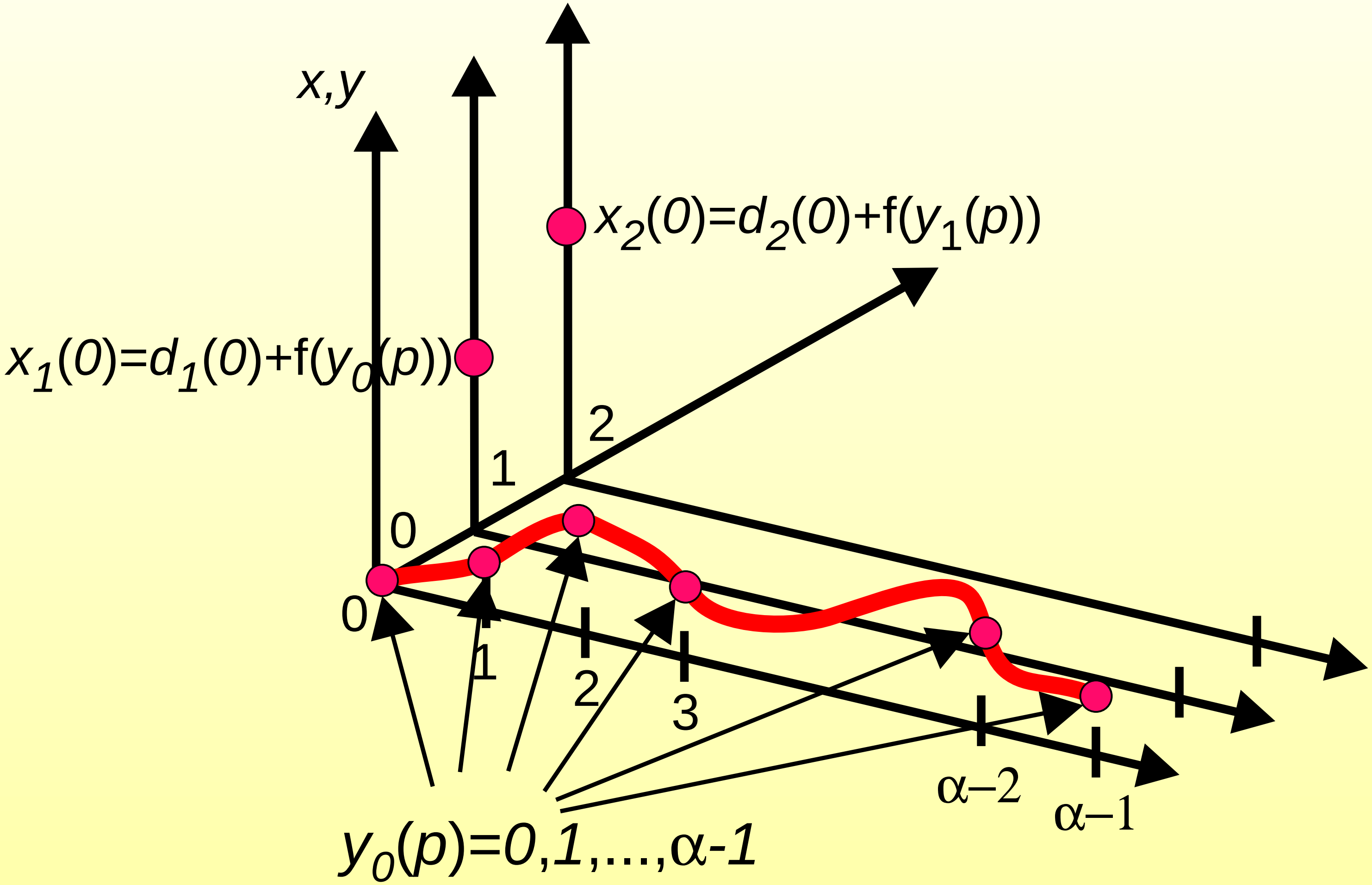
initial conditions:

$$\begin{aligned} x_{k+1}(0) &= d_{k+1}, : k \geq 0 \\ y_0(p) &= y(p), : 0 \leq p \leq \alpha - 1 \end{aligned}$$

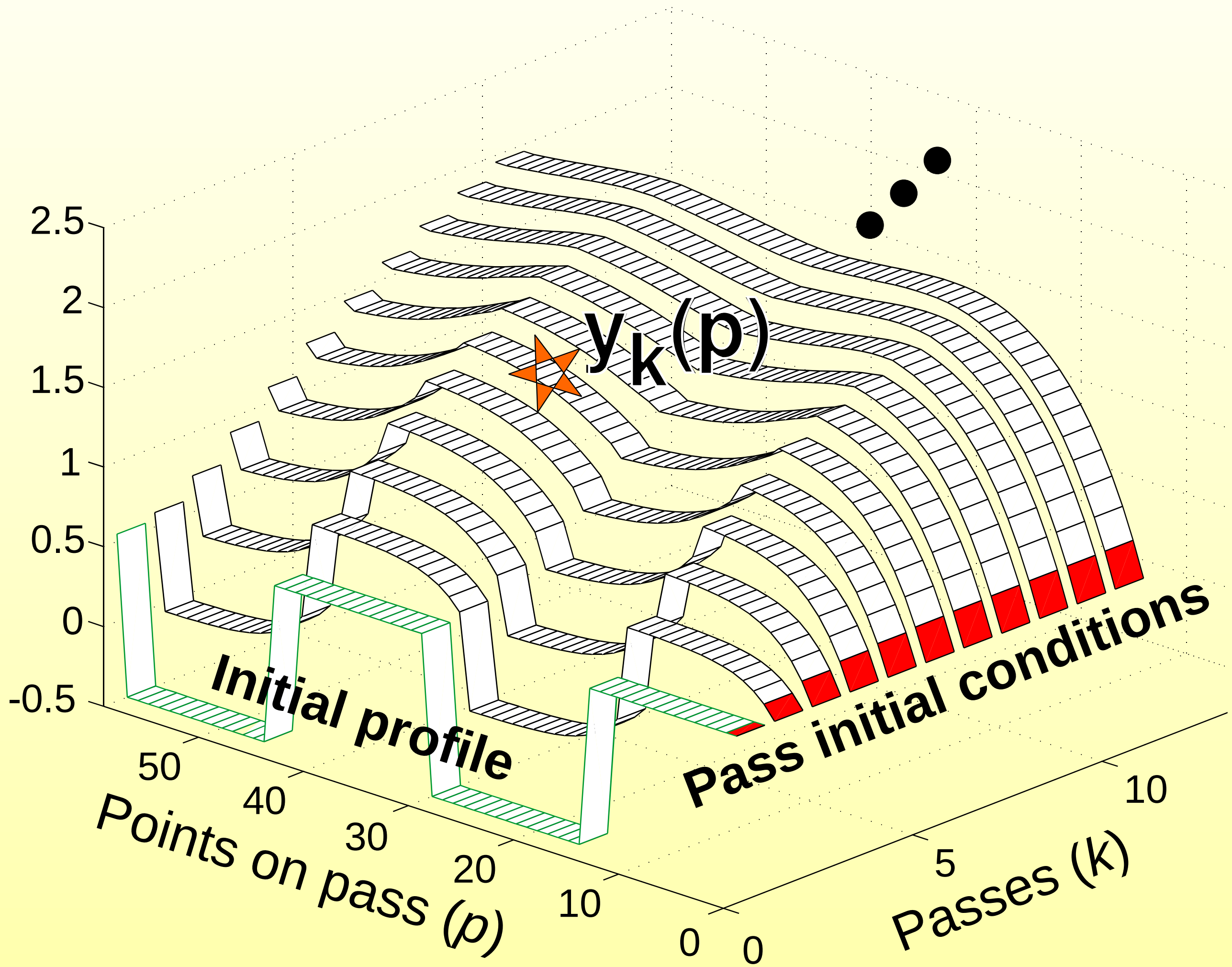
Static boundary conditions



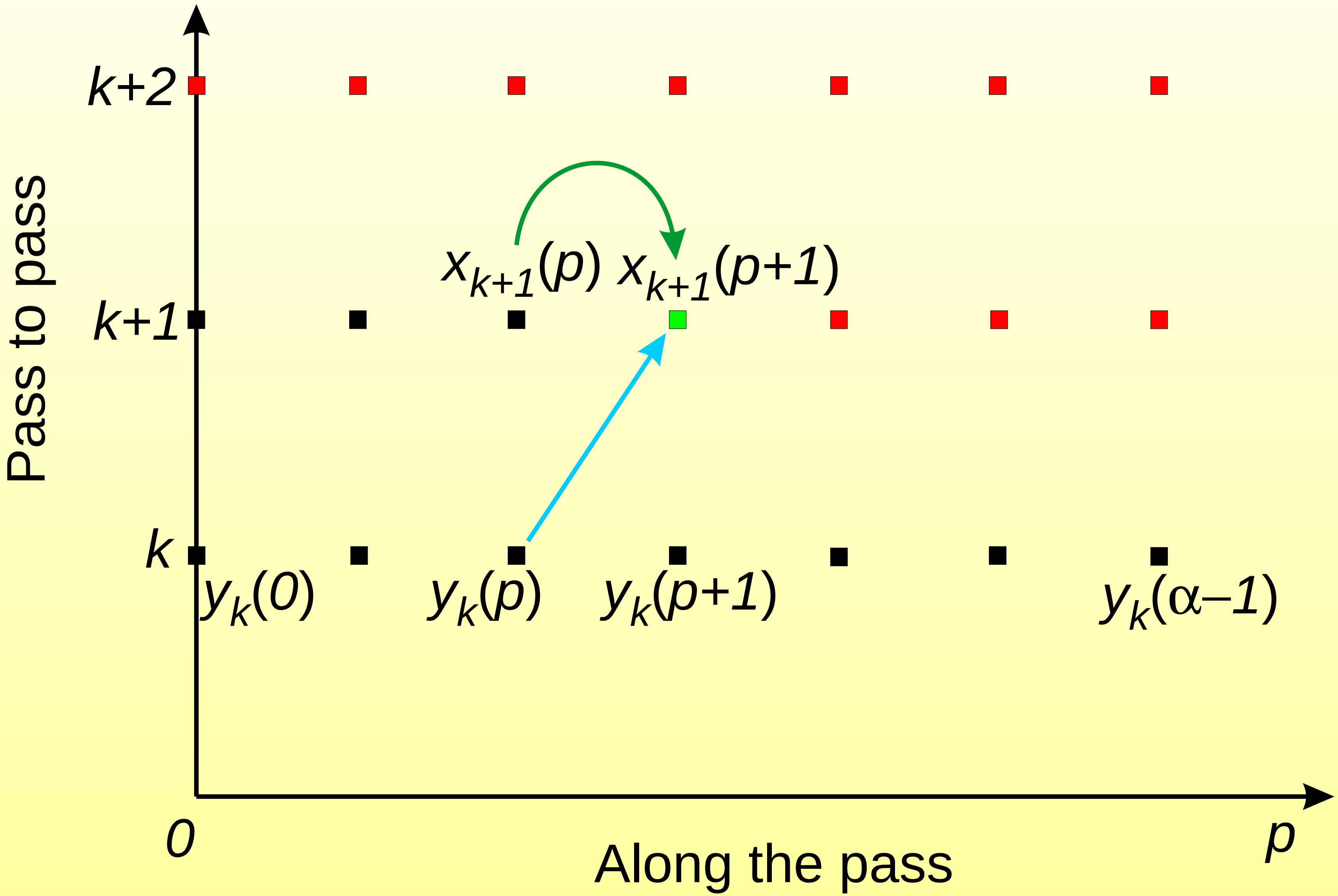
Dynamic boundary conditions



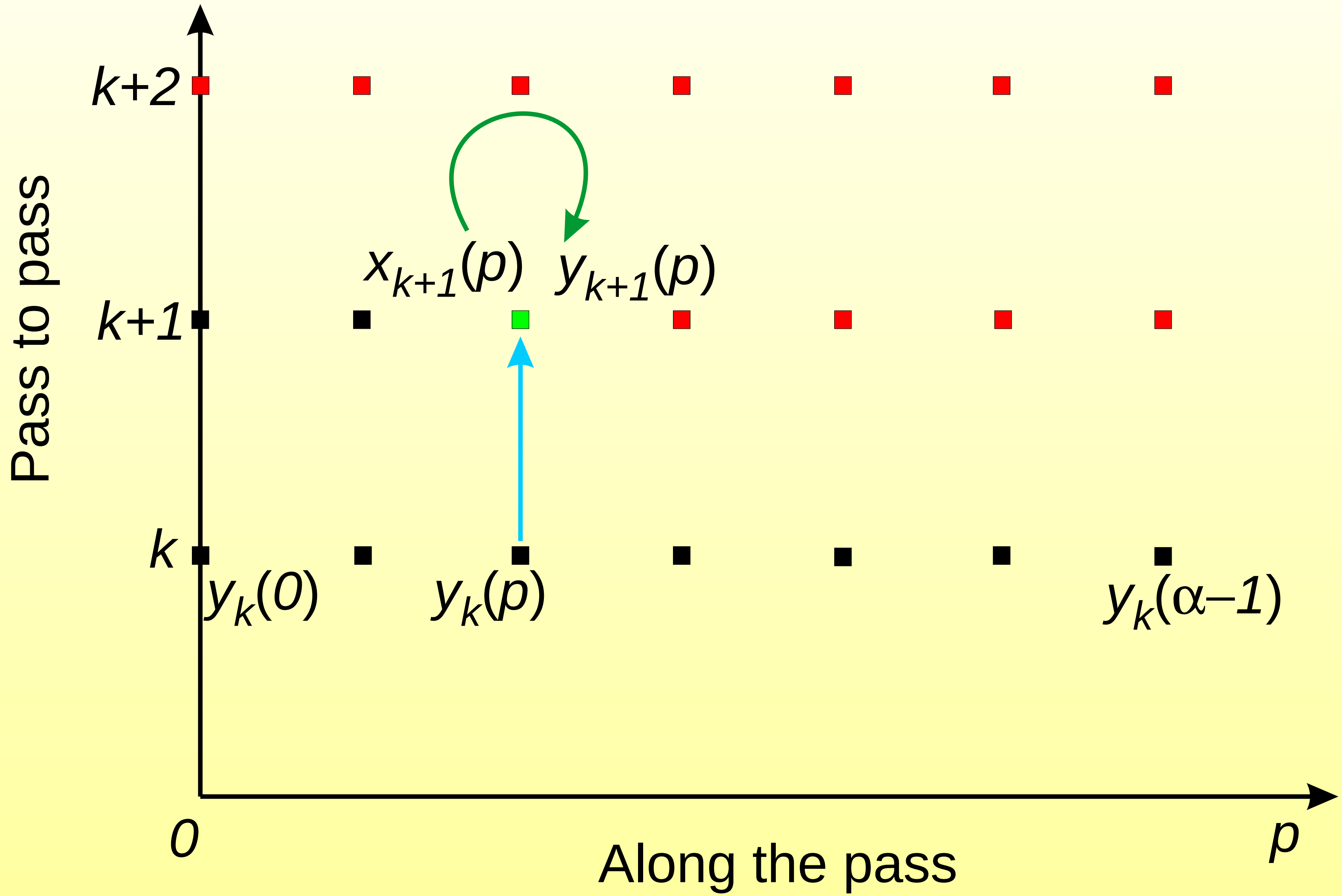
Linear repetitive processes — illustration



Linear repetitive processes — state



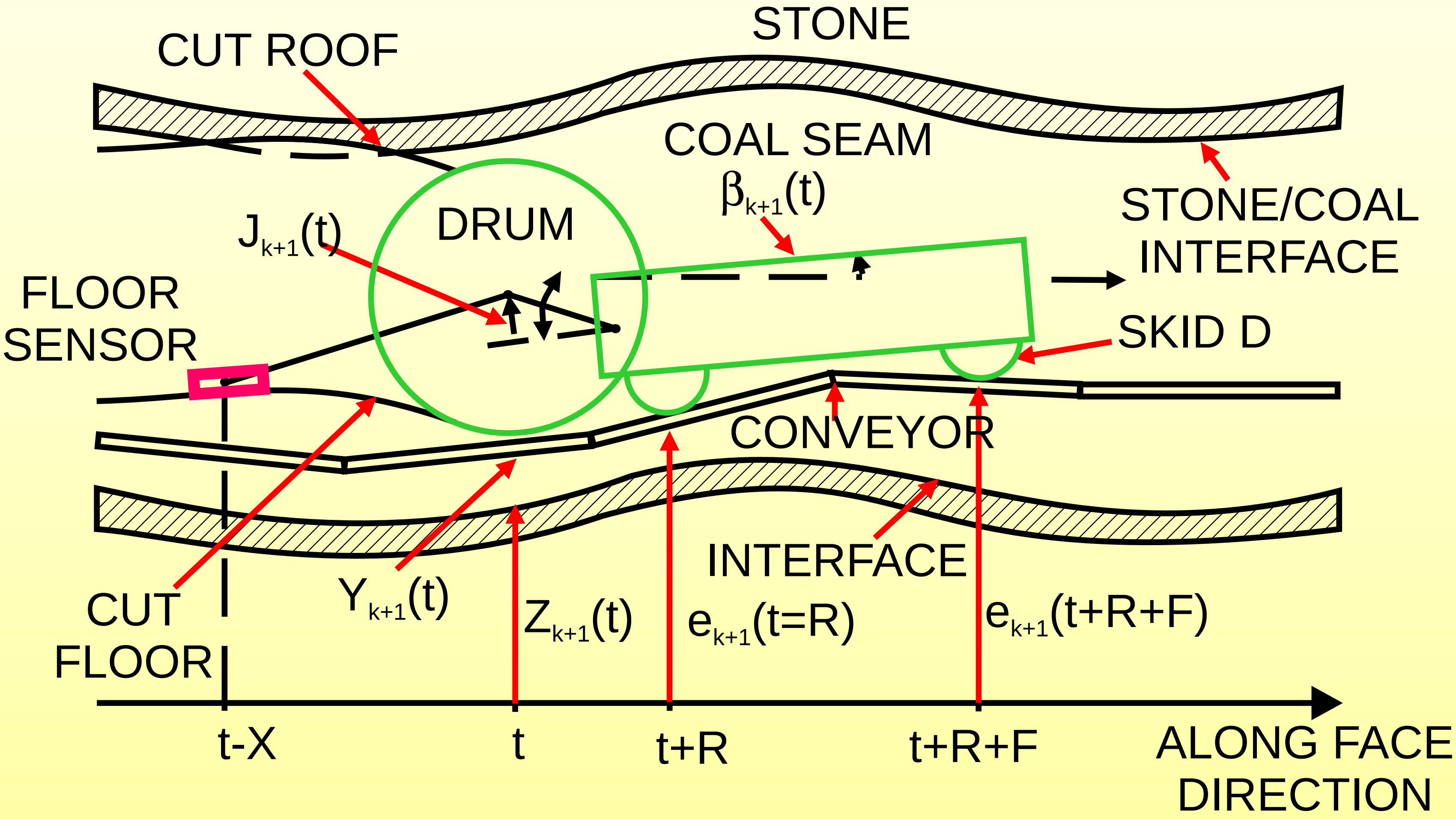
Linear repetitive processes — output



Applications

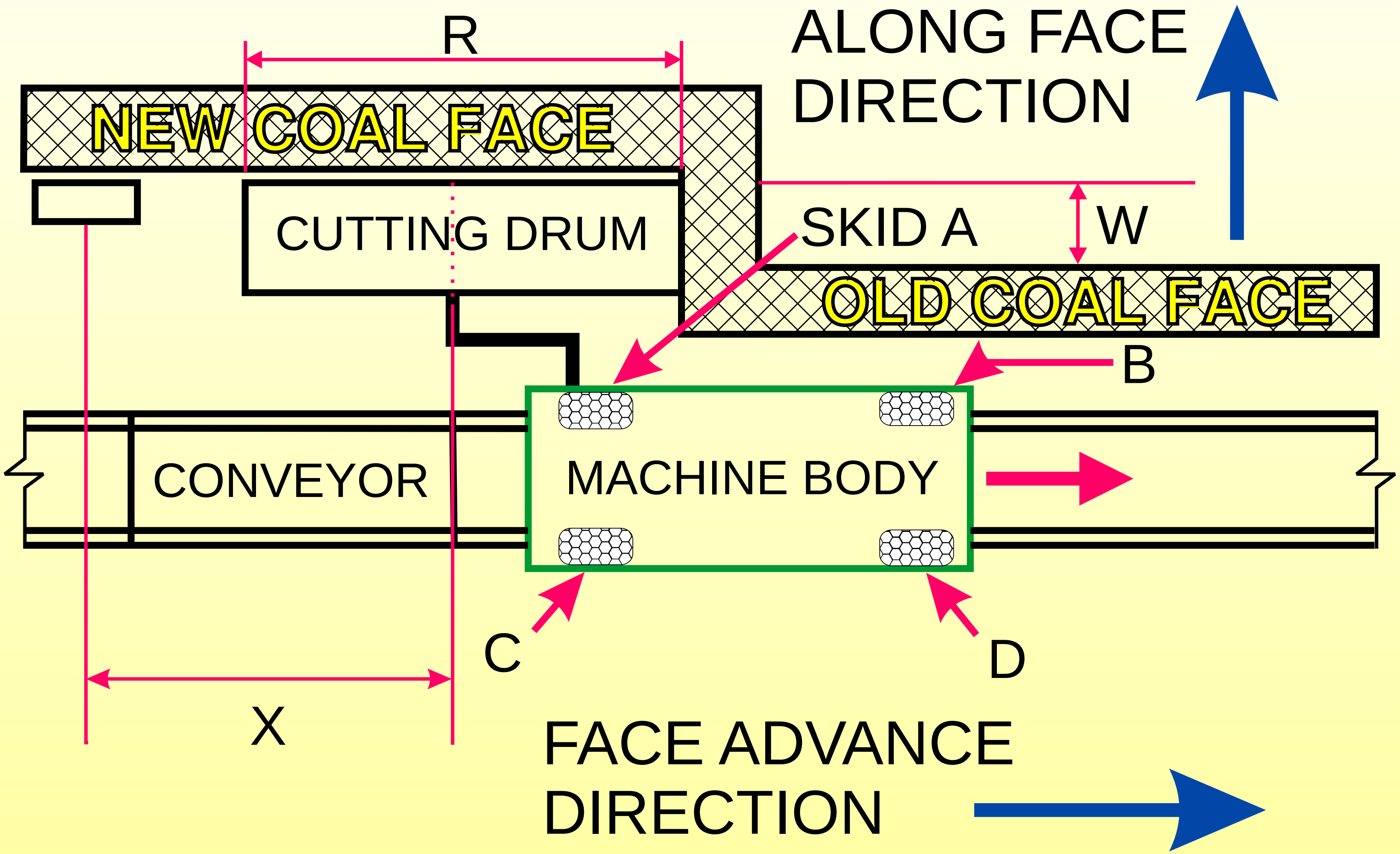
- Coal mining
- Web forming
- Metal rolling
- Iterative learning control

Coal mining

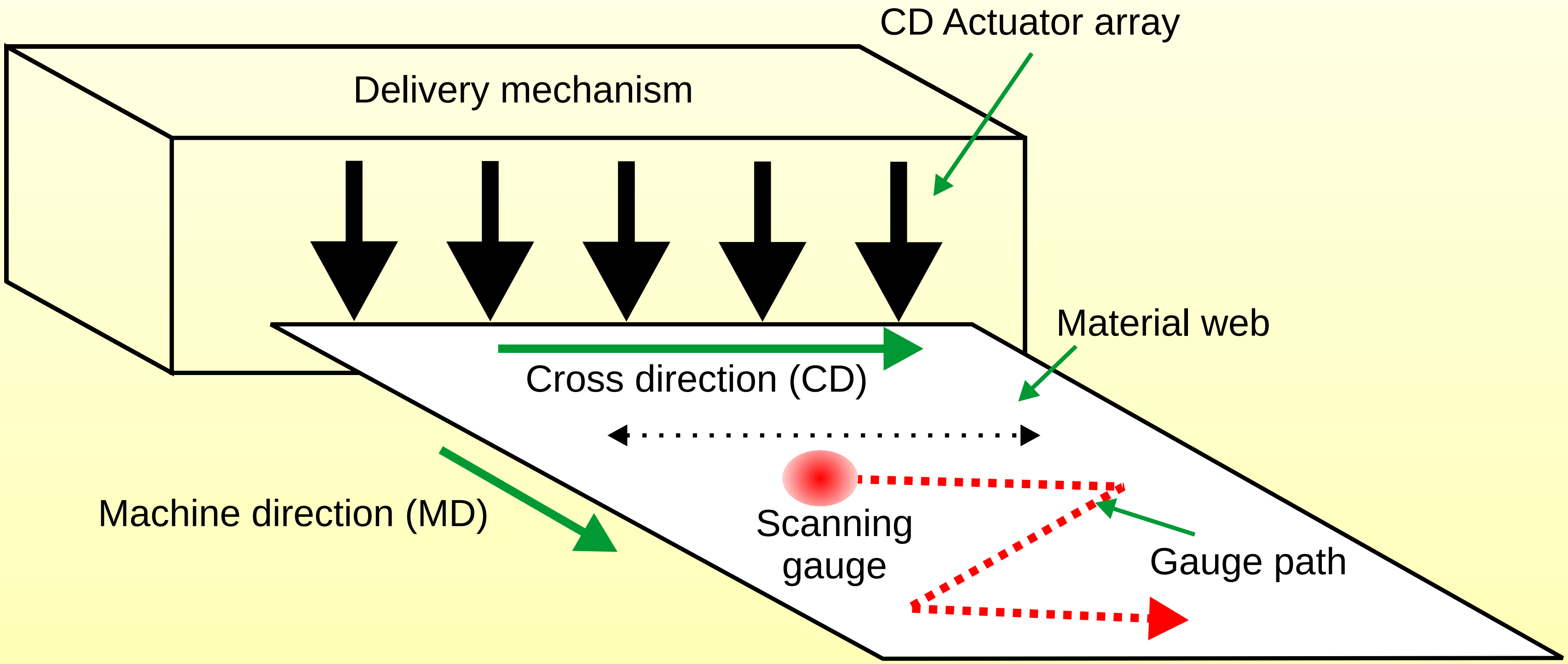


STONE
CUT ROOF
COAL SEAM
β k+1 (t)
STONE/COAL INTERFACE
DRUM
J k+1 (t)
FLOOR SENSOR
SKID D
CONVEYOR
INTERFACE
CUT FLOOR
Y k+1 (t)
Z k+1 (t)
e k+1 (t=R)
e k+1 (t+R+F)
t-X
t
t+R
t+R+F
ALONG FACE DIRECTION

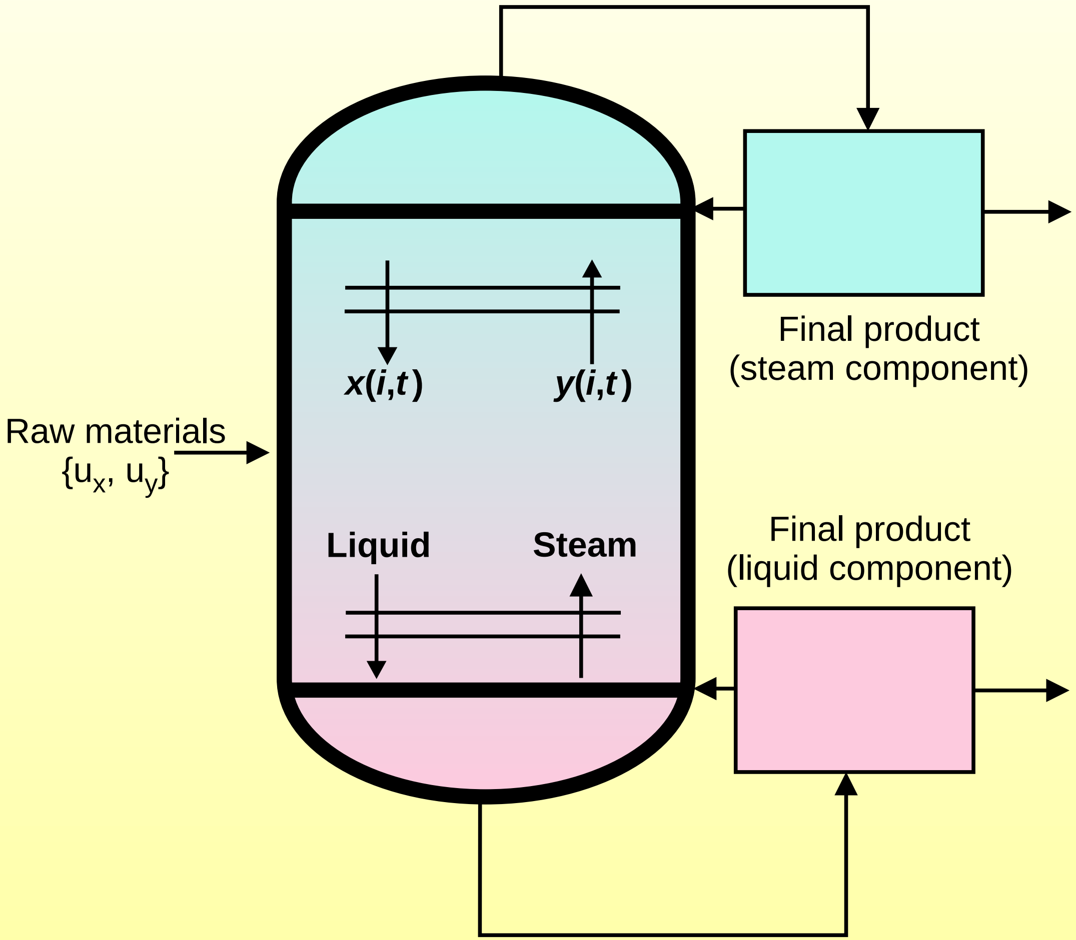
Coal mining — details



Web forming process



Rectification process of multiplicity mixtures in multiplate columns



$\{x(i,t), y(i,t)\}$ — unknown concentration of liquid and steam components of substance at i-th plate

Rectification process of multiplicity mixtures in multiplate columns — cont.

$$
\begin{aligned}
\frac{dx(i,t)}{dt} &= L_{i+1}(t)x(i+1,t) + L_i(t)x(i+1,t) \\
&\quad + R_i(x(i,t), y(i,t)) + u_x(i,t)
\end{aligned}
$$

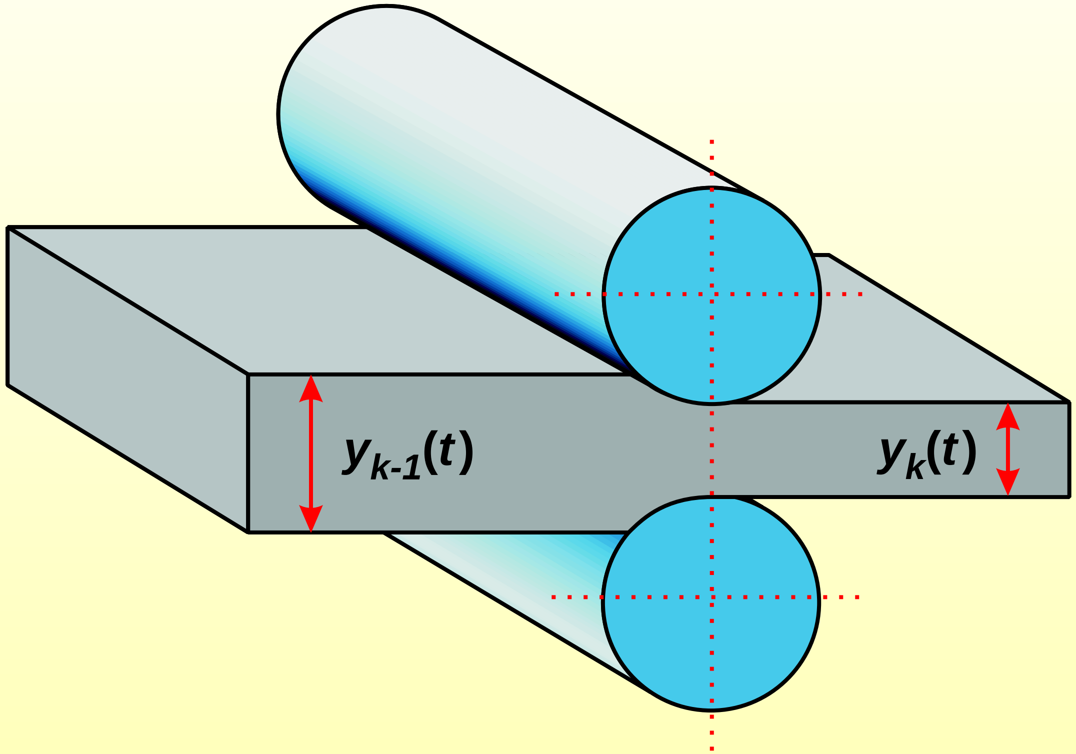
$$
\begin{aligned}
\frac{dy(i,t)}{dt} &= V_{i-1}(t)y(i-1,t) + V_i(t)y(i+1,t) \\
&\quad - R_i(x(i,t), y(i,t)) + u_y(i,t)
\end{aligned}
$$

L, V, R are given hydrodynamic background

For details see: Demidenko N.D.:

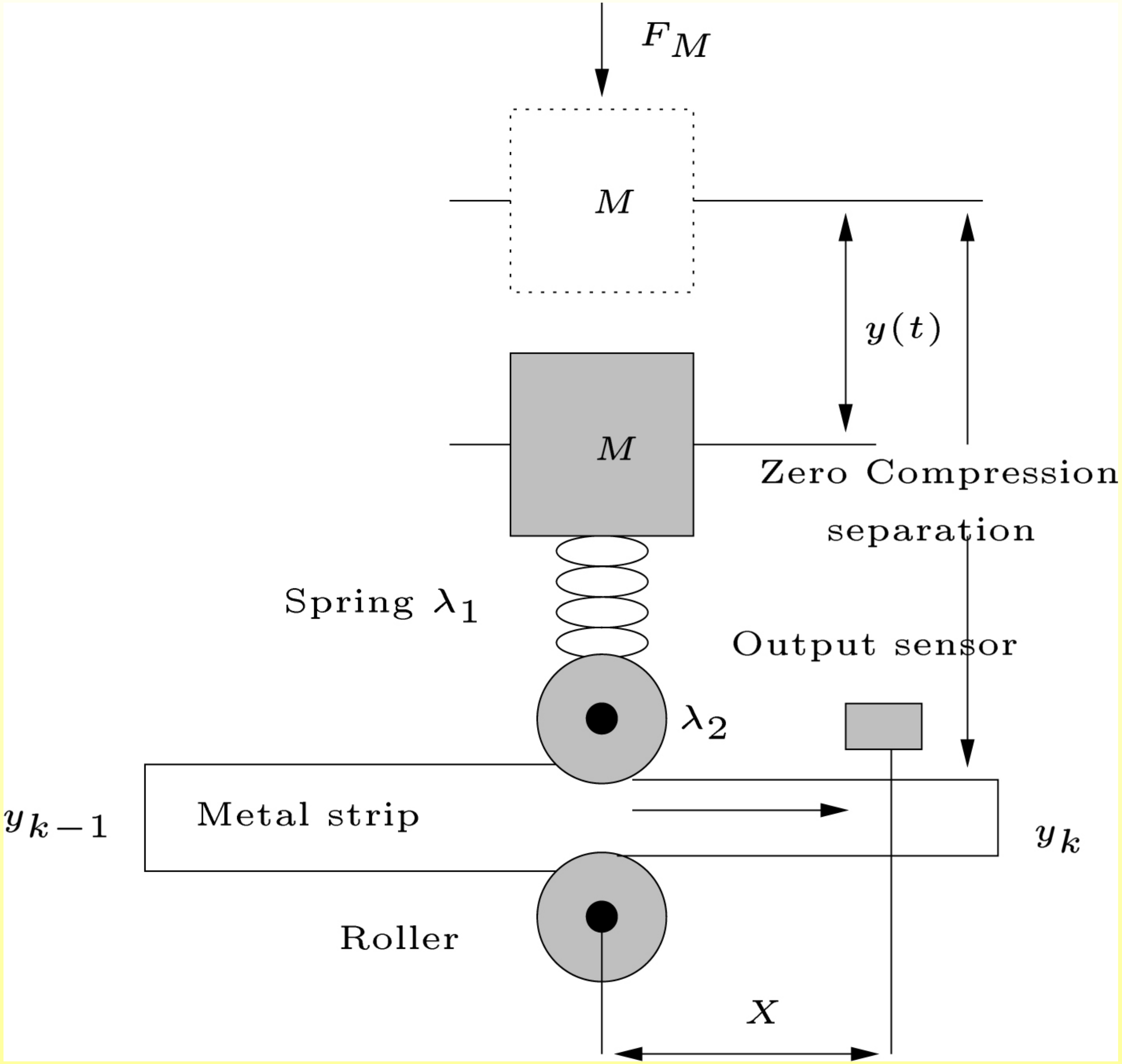
Simulation and optimization of heat and mass transfer process in chemical engineering , Moscow, Nauka, 1991, (in Russian)

Metal rolling



- y_{k-1} previous thickness of metal strip
- y_k current thickness of metal strip

Metal rolling — details



Metal rolling — model

$$\begin{aligned} x_{k+1}(p+1) &= A\,x_{k+1}(p) + Bu_{k+1}(p) + B_0 y_k(p) \\ y_{k+1}(p) &= C\,x_{k+1}(p) + Du_{k+1}(p) + D_0 y_k(p) \end{aligned}$$

$$\begin{aligned} u_k(p) &= F_M(p) \\ x_k(p) &= \left[y_k(p-1)\; y_k(p-2)\; y_{k-1}(p-1)\; y_{k-1}(p-2) \right]^T \end{aligned}$$

$$A = \begin{bmatrix} a_1 & a_2 & a_4 & a_5 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},\; B = \begin{bmatrix} b \\ 0 \\ 0 \\ 0 \end{bmatrix},\; B_0 = \begin{bmatrix} a_3 \\ 0 \\ 1 \\ 0 \end{bmatrix},$$

$$C = \left[\; a_1 \quad a_2 \quad a_4 \quad a_5 \;\right],\; D = b,\; D_0 = a_3.$$

Metal rolling — continued

where

$$a_1 = \frac{2M}{\lambda T^2 + M}, \;\; a_2 = \frac{-M}{\lambda^2 T + M}, \;\; a_3 = \frac{\lambda}{\lambda T^2 + M}\left(T^2 + \frac{M}{\lambda_1}\right),$$

$$a_4 = \frac{-2\lambda M}{\lambda_1(\lambda T^2 + M)}, \;\; a_5 = \frac{\lambda M}{\lambda_1(\lambda T^2 + M)}, \;\; b = \frac{-\lambda T^2}{\lambda_2(\lambda T^2 + M)}.$$

Metal rolling — notation

- $y_{k-1}(t)$ and $y_k(t)$ — thickness of the metal on the current and previous pass respectively,
- M — lumped mass of the roll-gap adjusting mechanism,
- λ_1 — the stiffness of the adjustment mechanism spring,
- λ_2 — the hardness of the metal strip,
- $\lambda = \frac{\lambda_1 \lambda_2}{\lambda_1 + \lambda_2}$ — the composite stiffness of the metal strip and the roll mechanism,
- $F_M(t)$ — the force developed by the motor,
- T — sampling period.

Iterative learning control

$$
\begin{aligned}
x(p+1) &= Ax(p) + Bu(p) \\
y(p) &= Cx(p)
\end{aligned}
\tag{1}
$$

Control aim:

$$
|y(p) - y_r(p)| < \epsilon \qquad p = 1, 2, \ldots, N
$$

Iterative control rule:

$$
u(p, k+1) = u(p, k) + \Delta u(p, k)
$$

2D case

(1) may be rewritten as

$$\begin{aligned} x(p+1,k) &= Ax(p,k) + Bu(p,k) \\ y(p) &= Cx(p) \end{aligned}$$

$$\begin{aligned} x(0,k) &= x_0 & k &= 0,1,\ldots \\ u(p,0) &= 0 & p &= 1,2,\ldots,N \end{aligned}$$

Control rule is said to be convergent iff

$$y(p,k) \to y_r(p), \quad p \in \{1,\ldots,N\}, \quad k \to \infty.$$

Control error

Let:

$$e(p,k) = y_r(p) - y(p,k).$$

Then

$$e(p,k+1) - e(p,k) = -CA\eta(p,k) - CB\Delta u(p-1,k)$$

where

$$\eta(p,k) \hat{=} x(p-1,k+1) - x(p-1,k).$$

Towards 2D model

Hence

$$\begin{bmatrix} \eta(p+1,k) \\ e(p,k+1) \end{bmatrix} = \left[\begin{array}{c:c} A & 0 \\ \hdashline -CA & I \end{array}\right] \begin{bmatrix} \eta(p,k) \\ e(p,k) \end{bmatrix} + \begin{bmatrix} B \\ -CB \end{bmatrix} \Delta u(p-1,k)$$

Finally, assumption

$$\Delta u(p,k) = Ke(p+1,k)$$

Final 2D model

leads to

$$\begin{bmatrix} \eta(p+1,k) \\ e(p,k+1) \end{bmatrix} = \left[\begin{array}{c|c} A & BK \\ \hline -CA & I-CBK \end{array}\right] \begin{bmatrix} \eta(p,k) \\ e(p,k) \end{bmatrix}$$

$$\begin{aligned} \eta(1,k) &= 0 \qquad k = 0,1,\ldots \\ e(p,0) &= y_r(p) - CA^p x_0 \qquad p = 1,2,\ldots,N \end{aligned}$$

convergence condition (asymptotic stability)

$$r(I - CBK) < 1$$

ILC — continuous case

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \qquad (2)$$

where

$$x \in \mathbb{R}^n, u \in \mathbb{R}^P, \{u(t), 0 \leq t \leq T\}.$$

Continuous case 2D approach

Aim of control

$$\sup_{0<t\leq T} |y(t) - y_r(t)| < \epsilon,\ 0 < t \leq T$$

(2) is modelled as

$$\begin{aligned} \tfrac{\partial x(t,k)}{\partial t} &= Ax(t,k) + Bu(t,k) \\ y(t,k) &= Cx(t,k) \end{aligned}$$

where k is the iteration number

Continuous case 2D approach — continued

Learning rule

$$u(t,k+1) \;=\; u(t,k) \;+\; \underset{\text{input modification}}{\underset{\uparrow}{\Delta u(t,k)}}$$

Initial conditions

$$\begin{aligned} x(0,k) &= x_0, \qquad & k &= 0,1,2,\ldots \\ u(t,0) &= u_0(t), \qquad & 0 &< t \le T \end{aligned}$$

Learning rule

Let:

$$\begin{aligned} e(t,k) &= y_r(t) - y(t,k) \\ \eta(t,k) &= \int_0^t \left[x(t,k+1) - x(t,k)\right] dt. \end{aligned}$$

Learning rule II

Then, one may show that

$$\frac{\partial \eta(t,k)}{\partial t} = A\eta(t,k) + B\int_0^t \Delta u(\tau,k)d\tau$$

and

$$e(t,k+1) - e(t,k) = -CA\eta(t,k) - CB\int_0^t \Delta u(\tau,k)d\tau.$$

Learning rule III

Assume that $y_r(t)$ is differentiable

$$\Delta u(t,k) = K\frac{\partial e(t,k)}{\partial t}.$$

The continuous - discrete Roesser model

$$\begin{bmatrix} \dfrac{\partial \eta(t,k)}{\partial t} \\ e(t,k+1) \end{bmatrix} = \left[\begin{array}{c|c} A & BK \\ \hline -CA & I - CBK \end{array}\right] \begin{bmatrix} \eta(t,k) \\ e(t,k) \end{bmatrix}$$

where

$$\begin{aligned} \eta(0,k) &= 0, \;\; k = 0, 1, \ldots \\ e(t,0) &= y_r(t) - Ce^{At}x_0 - \int_0^t Ce^{A(t-\tau)}Bu_0(\tau)d\tau \end{aligned}$$

convergence condition (asymptotic stability)

$$r(I - CBK) < 1$$